GMAT MATH FORMULA SHEET

ISBN 978-1-60743-439-9

NUMBERS

FRACTIONS TO DECIMALS

$\frac{1}{2} = 0.5$	$\frac{1}{5} = 0.2$	$\frac{1}{6} = 0.1\overline{6}$
$\frac{1}{3} = 0.\overline{3}$	$\frac{2}{5} = 0.4$	$\frac{5}{6} = 0.8\overline{3}$
$\frac{2}{3} = 0.\overline{6}$	$\frac{3}{5} = 0.6$	$\frac{1}{8} = 0.125$
$\frac{1}{4} = 0.25$	$\frac{4}{5} = 0.8$	$\frac{3}{8} = 0.375$
$\frac{3}{4} = 0.75$		$\frac{5}{8} = 0.625$
		$\frac{7}{8} = 0.875$

Know the decimal to fraction conversion for the same set of numbers, i.e. $0.375 = \frac{3}{8}$.

POWERS OF NUMBERS

$2^2 = 4$	$11^2 = 121$	$3^3 = 27$
$2^3 = 8$	$12^2 = 144$	$3^4 = 81$
$2^4 = 16$	$13^2 = 169$	$4^3 = 64$
$2^5 = 32$	$14^2 = 196$	$4^4 = 256$
$2^6 = 64$	$15^2 = 225$	$5^3 = 125$
$2^7 = 128$	$16^2 = 256$	
$2^8 = 256$	$17^2 = 289$	

Also know the equalities above in reverse order, i.e. $64 = 2^6$.
Notice the following:

$16^2 = (2^4)^2 = 2^{4\times 2} = 2^8 = 256$

$4^3 = (2^2)^3 = 2^{2\times 3} = 2^6 = 64$

$4^4 = (2^2)^4 = 2^{2\times 4} = 2^8 = 256$

OTHER NUMBERS

$\pi \approx 3.14$	$0! = 1$	$4! = 24$
$\sqrt{2} \approx 1.4$	$1! = 1$	$5! = 120$
$\sqrt{3} \approx 1.7$	$2! = 2$	$6! = 720$
$\sqrt{5} \approx 2.2$	$3! = 6$	

Notice the following:

$1.4^2 = 1.96 \approx 2$ $\quad 1.7^2 = 2.89 \approx 3$

$1 million = 1,000,000 = 10^6$

$1 billion = 1,000,000,000 = 10^9$

Prime numbers less than 20:

2, 3, 5, 7, 11, 13, 17, 19

ARITHMETIC

ORDER OF OPERATIONS

1. Parentheses.
2. Exponents.
3. Multiplication and Division from left to right.
4. Addition and Subtraction from left to right.

FRACTIONS

Adding Fractions

$$\frac{a}{b}+\frac{c}{d}=\frac{a\times d}{b\times d}+\frac{c\times b}{d\times b}=\frac{a\times d+c\times b}{b\times d}$$

Ex: $\frac{2}{3}+\frac{3}{4}=\frac{2\times 4}{3\times 4}+\frac{3\times 3}{4\times 3}=\frac{2\times 4+3\times 3}{4\times 3}=\frac{17}{12}$

Subtracting Fractions

$$\frac{a}{b}-\frac{c}{d}=\frac{a\times d}{b\times d}-\frac{c\times b}{d\times b}=\frac{a\times d-c\times b}{b\times d}$$

Ex: $\frac{1+y}{y}-\frac{x-1}{x}=\frac{x(1+y)}{xy}-\frac{y(x-1)}{yx}=\frac{x(1+y)-y(x-1)}{xy}$

$=\frac{x+xy-yx+y}{xy}=\frac{x+y}{xy}$

Multiplying Fractions

$$\frac{a}{b}\times\frac{c}{d}=\frac{a\times c}{b\times d}$$

Cancel any common factors from the denominators and the numerators and then multiply.

Ex: $\frac{\cancel{2}^{1}}{\cancel{3}}\times\frac{\cancel{3}^{1}}{\cancel{10}_{2}}=\frac{1\times 1}{2\times 2}=\frac{1}{4}$

Dividing Fractions

$$\frac{\frac{a}{b}}{\frac{c}{d}}=\frac{a}{b}\times\frac{d}{c}$$

Ex: $\frac{\frac{5}{8}}{\frac{3}{4}}=\frac{5}{\cancel{8}_{2}}\times\frac{\cancel{4}^{1}}{3}=\frac{5}{6}$

Multiplying/Dividing Fractions with Decimals

Convert decimals into fractions, and then calculate.

Ex: $0.33\times\frac{2}{11}\div 0.6=\frac{33}{100}\times\frac{2}{11}\div\frac{3}{5}=\frac{\overset{1}{\cancel{33}}}{\underset{20}{\cancel{100}}}\times\frac{\overset{1}{\cancel{2}}}{\underset{1}{\cancel{11}}}\times\frac{\overset{1}{\cancel{5}}}{\underset{1}{\cancel{3}}}=\frac{1}{10}$

MULTIPLYING/DIVIDING LARGE AND SMALL NUMBERS

Express the numbers using scientific notation. Then calculate.

Ex: $\frac{0.0004}{0.008}\times 500=\frac{\overset{1}{\cancel{4}}\times\overset{10^{-1}}{\cancel{10^{-4}}}}{\underset{2}{\cancel{8}}\times\underset{1}{\cancel{10^{-3}}}}\times 5\times 10^2=\frac{1\times 10^{-1}\times 5\times 10^2}{2}=25$

MULTIPLYING/DIVIDING EXPONENTIALS

ALGEBRA

EXPONENTS

Rule	Example

COMMON PRODUCTS

$(a+b)(c+d) = ac+ad+bc+bd$

Ex: $(2x+y)(3x-2y)$
$= 2x\cdot 3x+2x\cdot(-2y)+y\cdot 3x+y\cdot(-2y)$
$= 6x^2-4xy+3xy-2y^2$
$= 6x^2-xy-2y^2$

$(x+y)^2 = x^2+2xy+y^2$

Ex: $(5x+2y)^2 = (5x)^2+2\cdot 5x\cdot 2y+(2y)^2$
$= 25x^2+20xy+4y^2$

$(x-y)^2 = x^2-2xy+y^2$

Ex: $(\sqrt{5}-\sqrt{3})^2$
$= (\sqrt{5})^2-2\cdot\sqrt{5}\cdot\sqrt{3}+(\sqrt{3})^2$
$= 5-2\sqrt{15}+3 = 8-2\sqrt{15}$

$(x+y)(x-y) = x^2-y^2$

Ex: $(3-\sqrt{5})(3+\sqrt{5}) = 3^2-(\sqrt{5})^2 = 9-5 = 4$

Be able to apply the above rules in reverse order.

Ex: $x^2-2x-24 = (x+4)(x-6)$

$9y^2+4x^2+12xy = (3y+2x)^2$

$-8x+2x^2+8 = 2(-4x+x^2+4) = 2(2-x)^2$

$-9+4x^2 = 4x^2-9 = (2x+3)(2x-3)$

EQUATIONS WITH ONE VARIABLE

Cross Multiply Technique

If both the left hand side and the right hand side of the equation are or can be expressed as fractions, apply the cross multiply technique.

Ex: $\frac{x-3}{x+4}=\frac{2}{3}\Rightarrow 3(x-3)=2(x+4)$

$3x-9=2x+8$

$x=17$

Self check

Plug the solution you have found back into the original equation and check if the equation still holds.

Suppose $x=17$ is the solution you found for equation $\frac{x-3}{x+4}=\frac{2}{3}$.

$\frac{17-3}{17+4}=\frac{14}{21}=\frac{2}{3}$

The solution is correct.

SIMULTANEOUS LINEAR EQUATIONS

Solving by Substitution

1. From any of the two equations, write y in terms of x.
2. Plug the expression for y into the other equation.
3. Solve for x in the new one-variable linear equation.
4. Compute y by plugging the value of x into the expression found in step 2.

Ex: $\begin{cases} x+y=3 & (1) \\ x-y=1 & (2) \end{cases}$

Using equation (1) to write y in terms of x gives $y=3-x$. Plugging into equation (2) gives $x-(3-x)=1$. Solving x yields $x=2$. Plugging in the expression for y gives $y=1$.

ALGEBRA (CONTINUED)

EQUATION SOLVING CAUTIONS

One Equation with Two Unknowns

Two unknowns sometimes can be solved in one equation under certain constraints.

Ex: $9x + 5y = 52$

with x, y as positive integers.

The only possible solution is $x = 3$, $y = 5$.

Ex: Jay bought x hardcover books at \$9 each and y soft-cover books at \$5 each, spending \$52 in total. How many soft-cover books did he buy?

We get the same equation $9x + 5y = 52$. Be aware of the implicit assumption that x and y are positive integers.

COORDINATE

GEOMETRY

TRIANGLE

CIRCLE

GEOMETRY (CONTINUED)

ANGLE

MEASUREMENT

WORD PROBLEMS

RATE PROBLEMS

Two Objects Traveling in Opposite Directions

$$\text{Time} = \frac{\text{Total Distance Traveled}}{\text{Total Rate}}$$

where the total distance is the sum of the distance the two objects traveled, and total rate is the sum of the two rates.

Let D_T denote the total distance traveled, and R_T denote the total rate. We have the following three relations.

$$T = \frac{D_T}{R_T} \qquad D_T = R_T T \qquad R_T = \frac{D_T}{T}$$

Ex: Car A and Car B are 100 miles apart on along a route. Car A is traveling at a constant rate of 30 miles/hour, whereas car B is traveling at a constant rate of 20 miles/hour. If both cars are traveling toward each other, how long would it take for them to meet?

Let R_A and R_B denote the rates of car A and car B respectively, D_A and D_B denote the distance car A and car B traveled respectively. From the problem, we have the following:

$$R_A = 30 \qquad R_B = 20$$

$$D_T = D_A + D_B = 100$$

$$R_T = R_A + R_B = 30 + 20 = 50$$

$$T = \frac{D_T}{R_T} = \frac{100}{50} = 2$$

It would take them 2 hours to meet.

RATIO/PERCENTAGE PROBLEMS

WORD PROBLEMS (CONTINUED)

Relationship between Percentage and Ratio

Essentially, ratios and percentages convey the equivalent information. That is given the ratio between Q_1 and Q_2, we can find what percent Q_2 is of Q_1, what percent is Q_1 more/less than Q_2. Given Q_1 is x% more/less than Q_2, we can find the ratio of the two quantities.

Ex: If $Q_1:Q_2 = 3:4$, what percent is Q_1 less than Q_2?

$$\frac{Q_1}{Q_2} = \frac{3}{4} \Rightarrow Q_1 = \frac{3}{4}Q_2 = 0.75Q_2 = (1 - 0.25)Q_2$$

Q_1 is 25% less than Q_2.

Ex: If Q_1 is 10% more than Q_2, what is the ratio of Q_1 to Q_2?

$$Q_1 = (1 + 0.1)Q_2 = 1.1Q_2 = \frac{11}{10}Q_2 \Rightarrow \frac{Q_1}{Q_2} = \frac{11}{10}$$

The ratio of Q_1 to Q_2 is 11:10.

In data sufficiency problems, if a questions asks what percent one quantity is more/less than another, it is essentially asking for the ratio of the two quantities.

INTEREST RATE PROBLEMS

NUMBER PROPERTIES

DIVISIBILITY

PRIME NUMBER

EVEN & ODD

INEQUALITY

FACTORIAL

OTHERS

PROBABILITY AND COMBINATORICS

PROBABILITY

COMBINATORICS